

Quantum Mechanics of Neutrino Oscillations

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Abstract

Subtle problems in the theory of neutrino oscillations in vacuum are discussed [1]. It is shown that Lorentz invariance implies that in general flavor neutrinos in oscillation experiments are superpositions of massive neutrinos with different energies and different momenta. It is argued that a wave packet description of massive neutrinos is necessary in order to understand the physics of neutrino oscillations.

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1 Introduction

The standard theory of neutrino oscillations in vacuum [2] has been developed in the 70's on the basis of four main assumptions:

- (A1) Neutrinos are extremely relativistic particles.
- (A2) Neutrinos produced in charged-current weak interaction processes together with charged leptons α^+ are described by the flavor state

$$|\nu_\alpha\rangle = \sum_k U_{\alpha k}^* |\nu_k\rangle \quad (\alpha = e, \mu, \tau), \quad (1)$$

where U is the mixing matrix of the neutrino fields, $\nu_{\alpha L} = \sum_k U_{\alpha k} \nu_{k L}$. The massive neutrino states $|\nu_k\rangle$ are quanta of the fields ν_k of neutrinos with mass m_k . Their energy E_k and momentum p_k are connected by the relativistic dispersion relation

$$E_k^2 = p_k^2 + m_k^2. \quad (2)$$

Since in oscillation experiments neutrinos propagate along a macroscopic distance between production and detection, we consider only one spatial direction along the neutrino path.

- (A3) The massive neutrino states $|\nu_k\rangle$ have the same momentum p (“equal momentum assumption”, $p_k = p$), but different energies,

$$E_k = \sqrt{p^2 + m_k^2} \simeq p + \frac{m_k^2}{2p} = E + \frac{m_k^2}{2E}. \quad (3)$$

Here $E = p$ is the energy of a massless neutrino and the approximation is valid for extremely relativistic neutrinos.

(A4) The time T of propagation of neutrinos from source to detector is approximately equal to the source-detector distance L .

The massive neutrino states $|\nu_k\rangle$ evolve in time according to the Schrödinger equation, whose solution is

$$|\nu_k(t)\rangle = e^{-iE_k t} |\nu_k\rangle. \quad (4)$$

Using the assumptions (A1)–(A4) it is straightforward to derive the oscillation probability

$$P_{\nu_\alpha \rightarrow \nu_\beta}(T) = \left| \sum_k U_{\alpha k}^* e^{-iE_k T} U_{\beta k} \right|^2 \quad (5)$$

as a function of the time T and the oscillation probability

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* \exp\left(-i \frac{\Delta m_{kj}^2 L}{2E}\right) \quad (6)$$

as a function of the distance L , with $\Delta m_{kj}^2 \equiv m_k^2 - m_j^2$. The probability (6) is the one observed in real experiments where the propagation time T is not measured but the source-detector distance L is known.

Let us examine critically the assumptions (A1)–(A4).

The assumption (A1) is easily understood to be correct by noticing that, although not all existing neutrinos are extremely relativistic, in neutrino oscillation experiments one is interested in detectable neutrinos, which have energy larger than some fraction of MeV. Indeed, neutrinos are detected in:

1. Charged current weak processes which have an energy threshold larger than some fraction of MeV. For example¹:
 - $E_{\text{th}} = 0.233 \text{ MeV}$ for $\nu_e + {}^{71}\text{Ga} \rightarrow {}^{71}\text{Ge} + e^-$ in the GALLEX [3], SAGE [4] and GNO [5] solar neutrino experiments.
 - $E_{\text{th}} = 0.81 \text{ MeV}$ for $\nu_e + {}^{37}\text{Cl} \rightarrow {}^{37}\text{Ar} + e^-$ in the Homestake [6] solar neutrino experiment.
 - $E_{\text{th}} = 1.8 \text{ MeV}$ for $\bar{\nu}_e + p \rightarrow n + e^+$ in reactor neutrino experiments (for example Bugey [7] and CHOOZ [8]).
 - $E_{\text{th}} = 110 \text{ MeV}$ for $\nu_\mu + n \rightarrow p + \mu^-$.
 - $E_{\text{th}} \simeq m_\mu^2/2m_e = 10.9 \text{ GeV}$ for $\nu_\mu + e^- \rightarrow \nu_e + \mu^-$.
2. The elastic scattering process $\nu + e^- \rightarrow \nu + e^-$, whose cross section is proportional to the neutrino energy ($\sigma(E) \sim \sigma_0 E/m_e$, with $\sigma_0 \sim 10^{-44} \text{ cm}^2$). Therefore, an energy threshold of some MeV's is needed in order to have a signal above the background. For example, $E_{\text{th}} \simeq 5 \text{ MeV}$ in the Super-Kamiokande [9] solar neutrino experiment.

¹ In a scattering process $\nu + A \rightarrow B + C$ the squared center-of-mass energy $s = 2Em_A + m_A^2$ must be bigger than $(m_B + m_C)^2$, leading to $E_{\text{th}} = \frac{(m_B + m_C)^2}{2m_A} - \frac{m_A}{2}$.

On the other hand, although the direct experimental upper limits for the effective neutrino masses in lepton decays are not very stringent ($m_{\nu_e} \lesssim 3 \text{ eV}$, $m_{\nu_\mu} \lesssim 190 \text{ keV}$, $m_{\nu_\tau} \lesssim 18.2 \text{ MeV}$, see Ref. [10]), we know that the sum of the masses of light neutrinos ($m_\nu \lesssim m_Z/2 \simeq 45 \text{ GeV}$) that have a substantial mixing with ν_e , ν_μ and ν_τ is constrained to be smaller than about 5 eV by their contribution to the total energy density of the Universe [11].

The comparison of the cosmological limit on neutrino masses with the energy threshold in the processes of neutrino detection implies that detectable neutrinos are extremely relativistic. This is an important fact that is crucial for the theory of neutrino oscillations.

Let us consider now the assumption (A2). The flavor state $|\nu_\alpha\rangle$ in Eq. (1) is defined in order to be annihilated only by the flavor field $\nu_{\alpha L}$:

$$\langle 0 | \nu_{\alpha L} | \nu_\beta \rangle \propto \delta_{\alpha\beta}. \quad (7)$$

Let us check if this is true. The Fourier expansion of the left-handed components of the quantized fields of massive neutrinos is (see, for example, [12])

$$\nu_{kL}(x) = \int \frac{d^3\vec{p}}{(2\pi)^3 2E} \sum_{h=\pm 1} \left[a_k^{(h)}(\vec{p}) u_{kL}^{(h)}(\vec{p}) e^{-ip \cdot x} + b_k^{(h)\dagger}(\vec{p}) v_{kL}^{(h)}(\vec{p}) e^{ip \cdot x} \right], \quad (8)$$

where h is the helicity. The neutrino and antineutrino destruction operators $a_k^{(h)}(\vec{p})$, $b_k^{(h)}(\vec{p})$ satisfy the canonical anticommutation relations (for Majorana neutrinos $a_k^{(h)}(\vec{p}) = b_k^{(h)}(\vec{p})$)

$$\left\{ a_k^{(h)}(\vec{p}), a_j^{(h')\dagger}(\vec{p}') \right\} = \left\{ b_k^{(h)}(\vec{p}), b_j^{(h')\dagger}(\vec{p}') \right\} = 2E (2\pi)^3 \delta^3(\vec{p} - \vec{p}') \delta_{hh'} \delta_{kj}. \quad (9)$$

Hence,

$$\langle 0 | \nu_{kL}(0) | \nu_j(\vec{p}, h) \rangle = u_{kL}^{(h)}(\vec{p}) \delta_{kj}, \quad (10)$$

and we have²

$$\langle 0 | \nu_{\alpha L}(0) | \nu_\beta(\vec{p}, h) \rangle = \sum_{k,j} U_{\alpha k} U_{\beta j}^* \langle 0 | \nu_{kL}(0) | \nu_j(\vec{p}, h) \rangle = \sum_k U_{\alpha k} U_{\beta k}^* u_{kL}^{(h)}(\vec{p}) \propto \delta_{\beta\alpha}. \quad (11)$$

However, since detectable neutrinos are extremely relativistic, the contribution of neutrino masses in Eq. (11) can be neglected, leading to

$$\langle 0 | \nu_{\alpha L}(0) | \nu_\beta \rangle \propto \sum_k U_{\alpha k} U_{\beta k}^* = \delta_{\beta\alpha}, \quad (12)$$

because of the unitarity of the mixing matrix. Therefore, the flavor states (1) describe correctly neutrinos produced in weak interaction process only in the extreme relativistic approximation [13, 14], which is valid in neutrino oscillation experiments. Notice that the

² For simplicity, we consider a flavor neutrino with helicity h and definite momentum \vec{p} , according with assumption (A3). It is clear that the same conclusion is reached considering different momenta for the massive neutrino states.

flavor states (1) are not quanta of the flavor fields ν_α , only appropriate linear combinations³ of the massive neutrino states $|\nu_k\rangle$, quanta of the fields ν_k .

In order to discuss the physics of neutrino oscillations it is convenient to consider the simplest example of neutrino production: pion decay at rest,

$$\pi^+ \rightarrow \mu^+ + \nu_\mu. \quad (13)$$

The energy and momentum of each massive neutrino emitted in this process is determined by energy-momentum conservation [17]:

$$p_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 - \frac{m_k^2}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4m_\pi^2}, \quad (14)$$

$$E_k^2 = \frac{m_\pi^2}{4} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right)^2 + \frac{m_k^2}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) + \frac{m_k^4}{4m_\pi^2} \quad (15)$$

Since detectable neutrinos are extremely relativistic, only the first order approximation in the mass contribution is relevant:

$$p_k \simeq E - \xi \frac{m_k^2}{2E}, \quad E_k \simeq E + (1 - \xi) \frac{m_k^2}{2E}, \quad (16)$$

with

$$E = \frac{m_\pi}{2} \left(1 - \frac{m_\mu^2}{m_\pi^2}\right) \simeq 30 \text{ MeV}, \quad \xi = \frac{1}{2} \left(1 + \frac{m_\mu^2}{m_\pi^2}\right) \simeq 0.8. \quad (17)$$

Although the relations (16) have been derived in the specific case of neutrinos produced in pion decay at rest, they are valid in general for any process. Indeed, the first order approximation in the mass contribution must be proportional to m_k^2 because of the relativistic dispersion relation (2). In order to get a quantity of dimension energy, m_k^2 must be divided by E , that is the only available energy. The values of E and ξ are determined by the production process.

From Eq. (16) it is clear that in general the equal momentum assumption (A3) does not correspond to reality, unless one considers a special production process in which $\xi = 0$. However, as we will see in the following, the oscillation probability turns out to be independent of ξ . Hence, the incorrect equal momentum assumption ($\xi = 0$) leads to the correct transition probability (6).

How to take into account the different momenta of massive neutrinos in the derivation of the oscillation probability? The solution to this question starts from noticing that the oscillation probability should be Lorentz invariant because different observers measure the same flavor transition probability. But the probability (5) is not Lorentz invariant. In order to obtain a Lorentz invariant oscillation probability it is necessary to take into account not only the time evolution of the massive neutrino states, given in Eq. (4), which depends on their energy, but also their evolution in space, which depends on their momentum.

³ See Refs. [15, 16] for a different point of view.

The state $|\nu_k\rangle$ describes a massive neutrino at the production point $x = 0$ at the production time $t = 0$. The state that describes the same massive neutrino at the coordinate x at the time t is obtained by acting on $|\nu_k\rangle$ with the space-time translation operator $e^{-iP^\mu x_\mu}$:

$$|\nu_k(x, t)\rangle = e^{-iE_k t + i p_k x} |\nu_k\rangle. \quad (18)$$

Equations (1) and (18) lead straightforwardly to the Lorentz invariant oscillation probability

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, T) = \left| \sum_k U_{\alpha k}^* e^{i p_k L - i E_k T} U_{\beta k} \right|^2, \quad (19)$$

which depends on both energy and momentum of massive neutrinos. The probability (19) describes oscillations in space and time. In order to obtain the probability of oscillations in space it is necessary to express the time T in terms of the distance L .

In connection with the probability (19), it has been claimed recently by some authors [18, 19, 20] that the equal momentum assumption (A3) should be replaced by the “equal energy assumption”

(A3') The massive neutrino states $|\nu_k\rangle$ have the same energy $E_k = E$, but different momenta,

$$p_k = \sqrt{E^2 - m_k^2} \simeq E - \frac{m_k^2}{2E}. \quad (20)$$

This assumption could appear to be attractive because it leads to the vanishing of the time dependence of the probability (19), which becomes a probability of oscillations in space.

From Eqs. (16) and (17) one clearly see that in general the equal energy assumption is incompatible with energy-momentum conservation in the production process. Therefore, unless one considers a special production process in which $\xi = 1$, the equal energy assumption does not correspond to reality. However, since the oscillation probability turns out to be independent of ξ , the incorrect equal energy assumption ($\xi = 1$) leads to the correct transition probability (6), as well as the incorrect equal momentum assumption ($\xi = 0$).

There is another simple argument that shows that the equal energy assumption, as well as the equal momentum assumption, in general do not correspond to reality: Lorentz invariance implies that even if different massive neutrinos have the same energy (momentum) in one Lorentz frame, they have different energies (momenta) in all the other frames boosted along the neutrino propagation path [21].

Indeed, let us assume for example that in a Lorentz frame S different massive neutrinos have the same energy $E_k = E$, independent from the mass index k . In this frame the momenta of the massive neutrinos are given by Eq. (20).

In another Lorentz frame S' with velocity v with respect to S along the neutrino path the energy of the k^{th} massive neutrino is

$$E'_k = \gamma (E + v p_k) \simeq \gamma (1 + v) E - \gamma v \frac{m_k^2}{2E} = E' - \frac{v}{1 - v} \frac{m_k^2}{2E'}, \quad (21)$$

where $\gamma = (1 - v^2)^{-1/2}$ and $E' = \sqrt{\frac{1+v}{1-v}} E$ is the energy of a massless neutrino in S' . The difference between the energies of the k^{th} and j^{th} massive neutrinos in the frame S' is

$$\Delta E'_{kj} \equiv E'_k - E'_j = -\frac{v}{1-v} \frac{\Delta m_{kj}^2}{2E'}. \quad (22)$$

For relativistic velocities ($v \sim 0.1 - 1$), the energy difference is of the same order as the momentum difference,

$$\Delta p'_{kj} \equiv p'_k - p'_j = -\frac{1}{1-v} \frac{\Delta m_{kj}^2}{2E'}. \quad (23)$$

Therefore, it is clear that in the Lorentz frame S' the energies of different massive neutrinos are different and the equal energy assumption is untenable.

Is the transformation from a Lorentz frame S to a frame S' moving with relativistic velocity with respect to S important in practice? The answer is yes. Let us consider, for example, the simple case of pion decay (13). For the sake of illustration, let us consider the equal energy assumption to be valid for pion decay at rest, even if this assumption is incompatible with energy-momentum conservation, as discussed above. Then S is the Lorentz frame in which the pion is at rest.

Many experiments measure the oscillations of neutrinos produced by pion decay in flight. These are short and long baseline accelerator experiments and atmospheric neutrino experiments (see [22] for a review). The energy of the pions goes from a few hundred MeV (for example in the short baseline accelerator experiment LSND [23]) to hundreds of GeV (for example in the upward-going events measured in the Super-Kamiokande atmospheric neutrino experiment [24]).

It is clear that even if the equal energy assumption is valid for pion decay at rest, it cannot be valid even approximately in the case of short and long baseline accelerator experiments and atmospheric neutrino experiments. Indeed, considering for example a neutrino emitted in the forward direction by a pion decaying in flight with energy $E_\pi \simeq 200$ MeV, the laboratory frame S' is boosted with respect to the frame S in which the pion is at rest by a velocity $v \simeq 0.71$, which gives

$$\frac{v}{1-v} \simeq 2.4, \quad \frac{1}{1-v} \simeq 3.4. \quad (24)$$

From Eqs.(22) and (23) one can see that the energy and momentum difference between different massive neutrinos is of the same order of magnitude. Obviously, increasing the pion energy, the energy and momentum differences increase and tend to the same limit.

Let us emphasize that one would obtain the same result choosing another Lorentz frame in which the energies of different massive neutrinos are assumed to be equal: from Lorentz invariance the equal energy assumption cannot be simultaneously valid for all neutrino oscillation experiments in which neutrinos are produced by pion decay and it cannot be even valid in one experiment in which the decaying pion have a spectrum of energies (as always happens in practice).

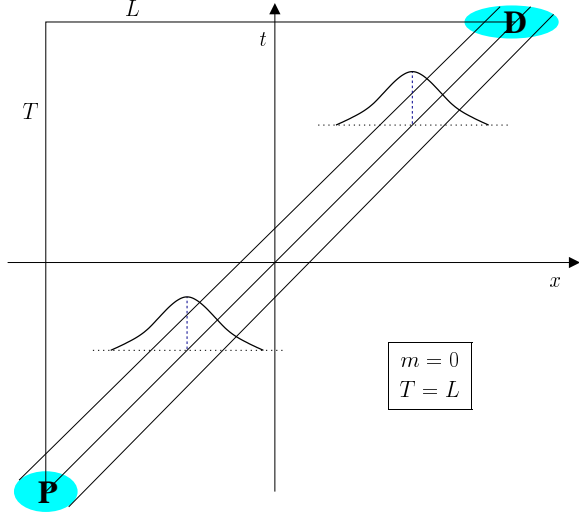


Figure 1

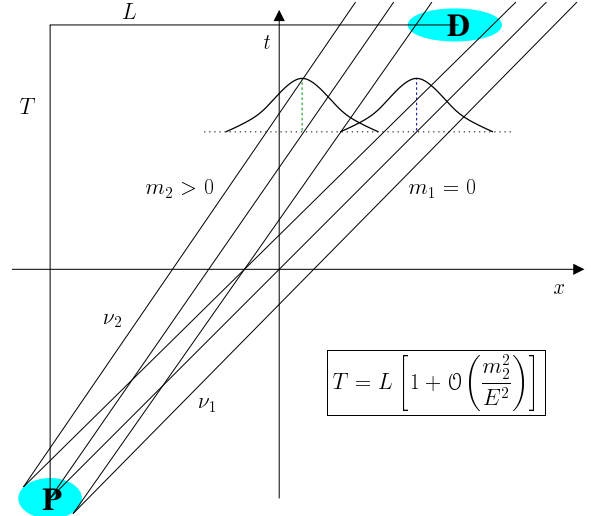


Figure 2

Another obvious problem of the equal energy assumption, as well as the equal momentum assumption, is the arbitrariness of the choice of the Lorentz frame in which it is valid, which is not based on any physical argument.

Since the equal energy and equal momentum assumptions are incompatible with Lorentz invariance and energy-momentum conservation in the production process, it is better to forget them and consider the different energies and momenta of the massive neutrinos given by energy-momentum conservation in the production process. Using the relativistic approximations in Eq. (16), the probability (19) becomes

$$P_{\nu_\alpha \rightarrow \nu_\beta}(L, T) = \sum_k |U_{\alpha k}|^2 |U_{\beta k}|^2 + 2\text{Re} \sum_{k>j} U_{\alpha k}^* U_{\beta k} U_{\alpha j} U_{\beta j}^* e^{-i\xi \frac{\Delta m_{kj}^2}{2E} L - i(1-\xi) \frac{\Delta m_{kj}^2}{2E} T}, \quad (25)$$

that describes oscillations in space and time, which depend on the characteristics of the production process through the quantity ξ .

In order to obtain the probability of oscillations in space, it is necessary to express the time T in terms of the source-detector distance L . This cannot be done considering massive neutrinos as plane waves, which extend over all space at all times. It is necessary to describe massive neutrinos as wave packets which are localized in the production region at the production time, propagate for a distance L during the time T and are localized in the detection region at the detection time.

The production, propagation and detection of a particle wave packet is illustrated by the space-time diagram in Fig. 1. The production and detection processes must be localized, but cannot be points in space-time, because in that case the uncertainty principle would imply that the energy and momentum have infinite uncertainty. Therefore, the production and detection processes occur coherently in finite space-time regions whose sizes are connected to the energy-momentum uncertainty by the uncertainty principle.

If the production process P occurs coherently in a finite space-time region of dimensions $(\Delta x_P, \Delta t_P)$, the emitted particle is described by a wave packet with size σ_x given by

$$\sigma_x^2 \sim \Delta x_P^2 + \Delta t_P^2, \quad (26)$$

as illustrated in Fig. 1, in which the space-time coherence region of the detection process has dimensions $(\Delta x_D, \Delta t_D)$. In Fig. 1 the propagating particle is assumed to be massless, leading to the relation $T = L$.

If only one particle propagates between the production and detection processes, as in Fig. 1, the propagating particle wave packet overlaps with both the production and detection processes. In this case the wave packet description does not lead to any interesting consequence and can be replaced by a plane wave treatment for all purposes except the derivation of the relation between T and L . On the other hand, in neutrino oscillation experiments the propagating neutrino is a coherent superposition of massive neutrinos, each described by a wave packet, as illustrated in Fig. 2 in the simplest case of two massive neutrinos with $m_1 = 0$ and $m_2 > 0$. The group velocity of each neutrino wave packet depends on its mass:

$$v_k = \frac{p_k}{E_k} \simeq 1 - \frac{m_k^2}{2E^2}. \quad (27)$$

Different wave packets are emitted simultaneously by the production process, but arrive at the detection process at different times

$$t_k = \frac{L}{v_k} \simeq L \left(1 + \frac{m_k^2}{2E^2} \right). \quad (28)$$

The time T in the phase

$$\Phi_{kj}(L, T) = -\xi \frac{\Delta m_{kj}^2}{2E} L - (1 - \xi) \frac{\Delta m_{kj}^2}{2E} T \quad (29)$$

of the interference term of the k^{th} and j^{th} massive neutrinos in the oscillation probability (25) must be averaged in the interval

$$\Delta T \sim [t_j, t_k] = L + \frac{\Sigma m_{kj}^2}{4E^2} L \pm \frac{\Delta m_{kj}^2}{4E^2} L, \quad (30)$$

when the wave packets of ν_k and ν_j overlap with the detection process (for $m_k > m_j$). In Eq. (30) we have defined $\Sigma m_{kj}^2 \equiv m_k^2 + m_j^2$. The value of the phase (29) in the time interval (30) is given by

$$\Phi_{kj}(L) \simeq -\frac{\Delta m_{kj}^2 L}{2E} - (1 - \xi) \frac{\Delta m_{kj}^2 L}{2E} \left(\frac{\Sigma m_{kj}^2 \pm \Delta m_{kj}^2}{4E^2} \right). \quad (31)$$

The second term on the right-hand side of Eq. (31) is strongly suppressed with respect to the first one because detectable neutrinos are extremely relativistic ($\Sigma m_{kj}^2 \ll E^2$ and $\Delta m_{kj}^2 \ll E^2$). Moreover, in real experiments the oscillations due to Δm_{kj}^2 are observable only

if $\Phi_{kj} \sim 1$ because of the average over the neutrino energy spectrum, the energy resolution of the detector and the source-detector distance uncertainty. Therefore, the leading term $-\Delta m_{kj}^2 L/2E$ must be of order one and the correction due to the second term on the right-hand side of Eq. (31) is negligible. In other words, if oscillations due to Δm_{kj}^2 are observable in a real experiment, the phase Φ_{kj} is practically constant in the time interval $[t_j, t_k]$:

$$\Phi_{kj} \simeq -\frac{\Delta m_{kj}^2 L}{2E}. \quad (32)$$

As one can see from Eq. (6), this is the standard value for the phase due to the interference of the k^{th} and j^{th} massive neutrinos. It is important here to notice that:

- A. The wave packet description of massive neutrinos is necessary to justify the approximation $T \simeq L$ (standard assumption (A4)) that leads to the standard phase (32).
- B. The quantity ξ has magically disappeared from the phase, thanks to the relativistic approximation and the approximation $T \simeq L$. This is very important because it implies that neutrino oscillations do not depend from the specific details of the production process.

As one can see from Fig. 2, another important consequence of the wave packet description is that the wave packets of different massive neutrinos propagate with different velocities and tend to separate (it is possible to show that the spreading of extremely relativistic wave packets is negligible). At a distance larger than the coherence length L_{kj}^{coh} the wave packets of the k^{th} and j^{th} massive neutrinos cannot both overlap with the detection process and the corresponding interference term is suppressed [25]. The coherence length L_{kj}^{coh} can be estimated by equating the separation of the wave packets,

$$|\Delta x_{kj}| = |v_k - v_j| T \simeq \frac{|\Delta m_{kj}^2|}{2E^2} L, \quad (33)$$

to the maximal separation allowed for interference,

$$|\Delta x|_{\text{max}}^2 \sim \sigma_x^2 + \Delta x_{\text{D}}^2 + \Delta t_{\text{D}}^2 \sim \Delta x_{\text{P}}^2 + \Delta t_{\text{P}}^2 + \Delta x_{\text{D}}^2 + \Delta t_{\text{D}}^2, \quad (34)$$

leading to

$$L_{kj}^{\text{coh}} \sim \frac{2E^2}{|\Delta m_{kj}^2|} |\Delta x|_{\text{max}}. \quad (35)$$

In conclusion we would like to remark that:

- I. The standard expression (6) for the neutrino oscillation probability in vacuum is robust.
- II. The relativistic approximation is crucial in order to obtain an oscillation probability that does not depend on the specific details of the production and detection processes.
- III. The oscillation probability must be Lorentz invariant because different observers measure the same flavor transition probability.

- IV. The equal momentum or energy assumptions do not correspond to reality. They are incompatible with Lorentz invariance and with energy-momentum conservation. We have shown that they are not needed for the derivation of the oscillation probability in space and time.
- V. The wave packet treatment is necessary in order to understand the physics of neutrino oscillations and the approximation $T \simeq L$ that allows to obtain the measurable oscillation probability in space from the Lorentz invariant probability of oscillations in space and time.

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